DOCUMENT RESUME

ED 461 501 SE 061 568

AUTHOR Wong, Regina M.; Lawson, Michael J.; Keeves, John
TITLE The Effects of Self-Explanation on Students' Problem-Solving in High School Mathematics.
PUB DATE 1998-03-00
NOTE 38p.; Contains small print.
PUB TYPE Reports - Research (143)
EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Foreign Countries; *Grade 9; High Schools; Independent Study; Mathematics Education; *Problem Solving; *Self Evaluation (Individuals); Student Attitudes; *Study Skills

ABSTRACT

The performance of a group of grade 9 mathematics students trained to use a self-explanation procedure during study was compared with that of students who used their typical study procedures. The processing activities used by the students during the study session and those used in a subsequent problem-solving test were observed. The focus of analysis was on the knowledge access, knowledge generation, management, and elaboration activities used by students. The self-explanation group showed more frequent use of each type of activity and also obtained higher scores on the problem-solving test. The difference in posttest performance of the groups was greatest on a set of far transfer items. Of particular note was the carryover effect of self-explanation training on students' processing in a subsequent problem-solving session. The relationships among the processing activities, students' beliefs, prior knowledge, and posttest performance were examined using a partial least squares path analysis procedure. Use of the self-explanation method had an indirect effect on performance, this effect being mediated by associated knowledge access and knowledge generation activity. There was no direct effect of method on performance. The strongest predictor of performance was the level of knowledge generation activity. The students' prior knowledge measure had weak direct and indirect effects on performance. Appendixes include: code labels and descriptions of three major categories of events: an illustration of direct and indirect paths in written solutions of two students; and descriptions of manifest and latent variables used in path analysis. (Contains 31 references, 4 tables, and 2 figures.) (Author)

Reproductions supplied by EDRS are the best that can be made
from the original document.

BEST COPY AVAILABLE

ED 461 501

Running head: SELF-EXPLANATION AND MATHEMATICS PROBLEM SOLVING

The effects of self-explanation training on students' problem-solving in high school mathematics

Regina M. Wong, Michael J. Lawson and John Keeves

School of Education

Flinders University

Adelaide, South Australia

March 1998

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

M. Lawson

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

BEST COPY AVAILABLE

Abstract

The performance of a group of Grade 9 mathematics students trained to use a self-explanation procedure during study was compared with that of students who used their typical study procedures. The processing activities used by the students during the study session and those used in a subsequent problem-solving test were observed. The focus of analysis was on the knowledge access, knowledge generation, management and elaboration activities used by students. The self-explanation group showed more frequent use of each type of activity and also obtained higher scores on the problem-solving test. The difference in posttest performance of the groups was greatest on a set of far transfer items. Of particular note was the carryover effect of self-explanation training on students' processing in a subsequent problem-solving session. The relationships among the processing activities, students' beliefs, prior knowledge and posttest performance were examined using a partial least squares path analysis procedure. Use of the self-explanation method had an indirect effect on performance, this effect being mediated by associated knowledge access and knowledge generation activity. there was no direct effect of method on performance. The strongest predictor of performance was the level of knowledge generation activity. The students' prior knowledge measure had weak direct and indirect effects on performance.

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

SCOU150

The effects of self-explanation training on students' problem-solving in high school mathematics.

The effects of different types of study activities on students' problem solving performance is of major interest to researchers and teachers. A recent focus of interest has been the types of explanations students develop as they undertake the study of textbook materials (e. g., Chi, Bassock, Lewis, Reimann & Glaser, 1989) and the impact of these self-explanations on problem solving. Chi et al. (1989) found that the more effective problem solvers in their sample were more active in developing generating explanations of the material being studied and in checking on the adequacy of their understanding. The study reported here is also concerned with the effects of use of these self-explanation procedures. In this case the focus is on the effects of self-explanation processes on the study processes and problem solving performance of high school students studying a new theorem in geometry. The principal objectives here were: (a) to find out if a less directive form of self-explanation training than that used to date would improve performance; and (b) to examine the influence of self-explanation training in a causal model that included both other variables predicted to influence performance and mediating processes argued to be associated with the self-explanation effect.

Focus on encoding processes

Interest in the effects of specific study procedures has been stimulated from a number of research directions. Chi and her colleagues (Chi, Bassock, Lewis, Reimann & Glaser, 1989; Chi & VanLehn, 1991; Chi, De Leeuw, Chiu & LaVancher, 1994) have proposed that research in this area is needed to illuminate the influence of encoding processes in problem solving, arguing that this needs to be distinguished from that played by later operating processes that transform, or proceduralize, the encoded representation into a cognitive skill. In the strongest form of this argument Chi et al. (1989, p. 146) contend that completeness of encoding has more influence than proceduralization on students' learning. In making this argument Chi et al (1989) have sought to point out the need to provide a more adequate account of the role of encoding in learning than has been included in models of skill acquisition such as that proposed by Anderson (1987).

Research on students' accessing and use of knowledge has also drawn attention to the influence of study procedures on problem solving. Recent research on high school students' problem solving in geometry has shown that some students do not access available knowledge at the appropriate time and do not necessarily effectively exploit the knowledge

that they do access (Lawson & Chinnappan, 1994). Chinnappan and Lawson (1994) argued that a major influence on access is the way that students establish and organise problem-relevant knowledge, a view that is reinforced by Glaser (1994): "Conditions of learning should foster increasing coherence and integration of knowledge." (p.353). These views on the key role of study procedures are, of course, also central to much of the research on cognitive strategy training.

The self-explanation effect

In Chi et al.'s (1989) initial work the more effective students were observed to study examples presented in the text by generating explanations that linked the new textual material, the examples worked out in the text and the students' existing knowledge. These students produced more task-related ideas in their self-explanations, more frequently related these ideas to the central principles of the domain of physics they were studying and also monitored their understanding more frequently. In subsequent discussion Chi has argued that during study use of the self-explanation process results in ongoing refinement of students' mental models relevant to the study topic, refinement that leads to more effective integration of knowledge(Chi et al., 1994).

The same self-explanation effect was observed in other research on learning of physics (Fergusson-Hessler & de Jong, 1990) and computer programming (Pirolli & Bielaczyc, 1989). The process of self-explanation was seen to enable students both to construct links between new information and existing knowledge and to generate new knowledge (Chi, DeLeeuw, Chiu & LaVancher, 1994). More recently Renkl (1997) investigated individual differences in self-explanation processes in the probability learning of university students, finding that the quality of the self-explanation was a significant predictor of learning gains. Renkl's findings reinforced those obtained in other self-explanation research: The most successful learners used more principle-based explanations, linked goals and actions more explicitly and were more anticipative in reasoning about examples.

These studies have shown that the more effective problem solvers generate more frequent and more powerful self-explanations while they are encoding the new study material and working through the examples provided. A question of interest in the current study is whether the self-explanation procedures also have the effect of improving access and use of problem-relevant knowledge beyond the period of study. In existing research where self-explanation has been the primary focus, the use of self-explanations during study has been clearly documented, but there has been no detailed analysis of processes being used by students as they undertake problem solving outside the study, or the training, situations. This issue is pursued in this study. Students' problem-solving performance is examined

when they are using encoded knowledge in a class test that occurred some time after study of a new section of the mathematics curriculum occurred.

Training in self-explanation

Observational research on the self-explanation effect has now been followed by studies with an instructional purpose. Chi et al. (1994) compared the performance of a group of eighth grade students prompted to use a self-explanation procedure with that of a group of students who read the same biology text. The self-explanation group made a greater gain from pretest to posttest than did the control group and frequency of self-explanation was associated with better understanding. In the posttest the performance advantage associated with self-explanation training was most apparent on the more difficult Category 3 and 4 questions. Bielaczyc, Pirolli and Brown (1995) trained university students in use of both self-explanation and self-regulation strategies as they studied a series of lessons on computer programming. The trained students showed greater problem-solving performance gains than did a group of control students, these gains being accompanied by significantly greater frequency of use of strategies that both elaborated and linked main ideas within the study text and connected ideas within the text to the examples used to illustrate programming procedures. In designing this study Bielaczyc et al. (1995) included both self-explanation and self-regulation strategies in their training procedure and did not seek to establish any estimates of the separate influences of these two different types of activity. In this study an attempt will be made to provide separate estimates of the influence of self-explanation and self-regulation, or self-management, activity.

Several other issues arise when the applicability of the findings of these instructional studies to the situations of high school students is considered. The first of these concerns the nature of the study procedures used by students. Chi et al(1994) were quite directive in their interaction with the students who were learning about the circulatory system using a self-explanation procedure. In addition to the prompts that were provided to induce explanation processes, students were required to clarify any statements that were judged to be vague and were asked to explain the functions of 22 key knowledge components that were distributed throughout the passage. Although these interactions played an important role in the explication of the effects of an explanation process they are not likely to be typical characteristics of a student's individual study session. A further point of interest in the current study is whether a less directive form of prompting of self-explanation, one more akin to that which could be used in individual study sessions, would also have a beneficial effect on students' problem solving on tasks that covered a range of difficulty.

The influence of prior knowledge

A further issue of interest in the current study concerned the influence of the students prior knowledge on performance following self-explanation training. On this issue the self-explanation literature is less clear. Chi et al. (1994, p. 475) note that the self-explanation effect "seems to be independent of ability (or achievement) and prior domain-relevant knowledge (as assessed in the pretests)". Chi draws a distinction between the influence of this rather narrowly prescribed domain-relevant knowledge mentioned above and that of "general world" or "commonsense" knowledge (Chi & VanLehn, 1991; Chi et al. 1994). In some of the examples cited by Chi and VanLehn (1991) the commonsense knowledge also seems to be clearly domain-related, so it is not clear whether the effect observed by Chi et al. (1994) is one that argues against the role of prior knowledge.

When self-explanation is seen as a type of knowledge construction activity it might reasonably be predicted that the outcome of self-explanation should be affected by the student's store of domain-relevant knowledge, since the more extensive and powerful that store the more potential there is for the construction of useful links between new material and prior knowledge. Related research also suggests that effects of prior knowledge on self-explanation use should be predicted. In research on the effects of training in use of elaborative-interrogation strategies, which also require students to generate explanations, prior knowledge has been shown to have very strong effects (e. g.,. Woloshyn, Pressley & Schneider, 1992). A further question of interest in this current study was, therefore, the degree of influence on performance of prior knowledge, relative to that of self-explanation training.

Students' beliefs

A final issue taken up in this study concerned the role of students' beliefs on their study and problem solving performance. To a large extent the role of affective factors is ignored by mainstream cognitive psychology. Yet, much other educational research, such as that concerned with self-efficacy (e. g.,. Schunck, 1983, Zimmerman, 1989) draws attention to the influence of students' expectations on their task involvement and problem solving. In the area of mathematical problem solving McCleod and Adams (1989) and Schoenfeld (1985)have argued strongly for recognition of the roe of students' expectations about mathematical tasks and about themselves as problem solvers. Schoenfeld (1985, p. 35) argued that " belief systems shape cognition" and mathematical behaviour, in some cases acting to prevent students from making progress on problems for which they have appropriate knowledge resources. The beliefs of the students could therefore be expected to

have an influence on the accessing and use of knowledge in problem situations, a topic discussed in an earlier section of this introduction.

Except for students' beliefs, the issues identified here as being associated with self-explanation have been considered in the self-explanation research literature. The consideration of these issues has, however, largely been discrete in the sense that the frequency of different types of processes and the influence of student characteristics have been considered in separate analyses. This type of analysis has been appropriate for the objectives set for the various studies and is given consideration here. However, it is also possible to consider the influence of a range of variables on performance in a different way, by conceptualising the array of variables as a causal model. From the above analysis a path model involving factors of prior knowledge, beliefs, instructional method, knowledge access and knowledge generation and self-regulation activity is presented and the degree of influence of these factors on problem-solving performance is estimated. The model is presented in Figure 1.

This study is designed to investigate whether a self-explanation effect would emerge in a study situation that was more typical of those used in regular classroom study and whether the self-explanation training would affect students' behaviour during problem solving and their performance. The procedures used in the study permitted the estimation of the influence of self-explanation training relative to that of variables proposed to influence problem solving performance. The variables of specific interest here are the students' beliefs about themselves and the task, their prior knowledge, their self-regulation activity and the mediating processes of knowledge access and knowledge generation observed in previous self-explanation research.

Method

Participants

Students drawn from Grade 9 classes in a suburban high school in Adelaide participated in the study: 26 were females and 21 males. The average age of the group was 14 years and 3 months. The classes from which the students were drawn were regarded as the top two mathematics classes of the ten classes at that grade level.

The students were ranked on the basis of their performance on class achievement tests and the member of each ranked pair was assigned at random to either the self-explanation group or the control group. Following completion of all activities in the study data were available for 47 students, with 24 students being in the explanation group and 23 students in the control group. There were 13 females in each group.

Procedure

Students participated in six sessions during the course of the study. In the first session students completed the Mathematical Beliefs Questionnaire (MBQ) which was adapted from the study by Hayes (1993). The MBQ contained 42 items presented as 5-point Likert scales and grouped into seven subscales that were concerned with: The nature of mathematical ability, performance goals, learning goals, styles of learning, the students' role, the general nature of mathematics and the nature of mathematical tasks.

A pretest of students' prior knowledge of plane geometry was completed in the second session. This test included 27 items taken from the students' textbook and covered knowledge of names of angles, triangles and parts of circles, knowledge of different types of angles and knowledge of theorems that had been studied previously and of the target theorem. The final items required use of these theorems in non-word and word-problem formats. These were areas that were of direct relevance to the content of the study material and to the posttest problems.

The third session involved each student in both groups in the study of the new theorem using the booklet prepared for the study. The text for study in shown in Figure 1. Each student worked alone with the researcher during this session. At the start of this session the student was trained in use of a think-aloud procedure following the recommendations set out in Ericsson and Simon (1993). Students were encouraged to talk all the time as they worked through the booklet and were given neutral prompts if they remained quiet for more than five seconds. Only a concurrent verbal report was sought and no retrospective account of problem solving activity was required. The student's written activity and talk were videotaped and audiotaped for later analysis. During the period of study students in both groups had access to normal classroom materials and to two reference sheets. One reference sheet listed and illustrated geometry theorems that had been studied in previous class work. The other sheet presented the meanings of symbols and terminology about the circle.

After instruction and practice in use of the think-aloud procedure the control group worked through the booklet using their typical procedures for studying a new section of the textbook. The students were asked to imagine that they were studying a new section of their textbook in order to understand the new theorem and were encouraged to use whatever procedures would help them with this task. Once students indicated that they understood the study procedure they began working through each page of the study booklet. The researcher provided neutral prompts reminding them to keep talking to provide a full report for the videotape.

The explanation group received the same think-aloud instructions and practice and then worked through each page of the booklet. On each of the pages for this group three questions were displayed at the top of the page. These were the questions that contained the self-explanation training. The questions were designed to encourage the students to analyse the text, to attempt to explain it to themselves, and to check their comprehension of the material. These activities are central to the notion of a self-explanation as a process of explaining instructional material to oneself during study. The questions were designed to require the student to pay attention to each section of the material, to link that with statements that had previously been presented and with what was already known in this area of problem solving, and to identify anything that was not yet understood. The form of the questions varied slightly between the different sections of the study material, as shown below.

Theorem section

1. What parts of this page are new to me?

2. What does the statement mean?

3. Is there anything I still don't understand?

Proof section

1. What parts of this page are new to me?

2. How does the new piece of information help with the proof of the theorem?

3. Is there anything I still don't understand?

Example section

1. What parts of this page are new to me?

2. How does the new piece of information help me to solve the sample problem?

3. Is there anything I still don't understand?

During the initial part of the explanation training students' attention was explicitly drawn to the questions. Reminders were given if students forgot to use the questions, but for most students use of the questions quickly became routine. Explanation group students received neutral prompts from the researcher if they were silent for more than five seconds. As with students in the control group, the prompts were reminders to keep talking in order that the concurrent protocol would be as full a report as possible.

The fourth session was a review session which was scheduled one week after all training had been completed. In this group session students in both groups followed through as the researcher reviewed each of the theorems on the theorem review sheet that now included a restatement and illustration of the target theorem. This session was scheduled on the day prior to the posttest in order that students who had studied the new material some days before would have reviewed the target theorem at the same time as students who had finished the third session more recently.

Session five was the posttest session in which students completed two sets of posttest items. Twelve items on this test were directly focused on use of the target theorem. Two of these items (Training items) were isomorphic to the worked example used in the study booklet and required only use of the target theorem for generation of a solution.. Five items involved minor surface changes to the format of the worked example in Figure 1, such as use of a full circle or re-orientation of the triangle to the bottom half of the circle. All of these Near Transfer items required use of the theorems that were presented in the study material and use of one previously studied theorem, the sum of angles in a triangle theorem. . The remaining five problems were classified as Far Transfer problems. These items all required use of at least one theorem additional to those included in the study booklet, or involved constructions that had not been included in the study booklet, or were presented as word problems.

A final section of the posttest included items taken from the pretest in order to assess whether there were indications that this part of the student's domain-specific knowledge had changed following study of the new material and completion of the posttest items involving use of that theorem. These items involved use of key terms, use of right angles in calculations and of theorems about the straight line theorem, isosceles triangles and the sum of angles in a triangle.

During the final session students were shown the results of all their previous tests and the self-explanation procedure was described for the benefit of the control students. All students were then provided with a written description and rationale for use of the procedure and a description of its expected benefits when used while they were studying new sections in their mathematics textbook.

Materials

The subject content for the study was drawn from the students' textbook and presented a theorem in geometry that the students had not previously studied. The section of the textbook that was studied comprised three sections: Statement of the theorem, illustration of the proof of the theorem and a worked example in which the theorem was used to calculate the value of another angle. The text to be studied is shown in Figure 1.

The text was presented to the students in a booklet with one new unit of input being presented on each page of the booklet. In general these units of input corresponded to lines of the text, though this was varied in certain cases. For example, the statement of the theorem was presented first and then the diagrams to the right of that box were presented on the next page. In other cases two lines of text were presented where they were necessary to make a meaningful unit of input.

Coding of verbal protocols of the study session

A transcript of each student's activity while engaged in study of the new text material in session three was prepared from the videotape. In coding these transcripts the major interest was in the extent to which students engaged in three broad classes of activity during their study of the new material:

Knowledge access. The statements or actions included in this category were ones that involved identification of any information not explicitly provided in the study text. The purpose of this code was to identify use of the student's prior mathematical knowledge and included retrieval of names, meanings of geometrical features, or of previously studied theorems or proofs. This code was also used for retrieval of information that had been generated previously during the study period.

Knowledge Generation. These statements or actions were concerned with the use of given information or accessed knowledge to create new connections or relationships. This might include drawing, labelling or extension of a diagram; calculations and algebraic manipulations; reasoning, hypothesising or justifying that involved application of the target theorem; or relating of that theorem to theorems that had been previously accessed.

Management. This was activity concerned with assessment of the current state of problem solving activity. This involved statements or actions such as: rereading of the text; identification of a text element as new; monitoring of comprehension; statement or re-stating of difficulties in comprehension; checking of the goal, of the accuracy of reasoning, calculation or other actions; and evaluation of the solution.

Codes for each of the statements or actions included in these three categories were defined and these are shown in Appendix 1. Two coders practised use of the coding scheme and then completed independent coding of sample transcripts. Interrater reliability for allocation of events to the coding categories was 88%. The measures used in subsequent analyses were frequencies of the three broad categories.

Scoring of posttest performance

The students' written solution attempts and the verbal protocols of their think-aloud reports while studying the text in session three provided different, but complementary, sets of information for analysis and were scored in three ways. First the response for each question was scored to recognise appropriate steps in the solution attempt. In some of the training items there was only one step required. In other items the student needed to make several moves in order to generate a solution and each of these moves was awarded one point. The score for an item was the sum of the scores for the moves identified as appropriate for the item. Item scores were summed to provide a total posttest score.

A second system of scoring of written solutions was used to provide information on the knowledge used by students during the solution attempt. This was done in two ways. Each instance of use of theorem knowledge necessary for the most direct path to the solution was given a point toward a *direct path* score. Other instances of use of related theorems during the solution attempt, that were not needed in the most direct path were awarded a point toward the *indirect path* score for the item. The difference between these two sets of scores can be illustrated with reference to the Far Transfer item shown in Appendix 2. For that item an economical solution would involve use of the target theorem and one additional theorem, the angle sum of a triangle theorem. With these two theorems the student could calculate the value of x. This solution path is to be contrasted with the solution of a student that involved use of such related knowledge as the straight line theorem, the equality of vertically opposite angles, and the equality of two sides and the base angles in an isosceles triangle. All of this related knowledge could have been used in developing a solution to the problem, though it was not needed to generate the most direct path. Use of this additional theorem knowledge was recognised in the indirect path score.

An elaboration score was the final score derived from inspection of students' post test written solutions. Elaboration activities are activities that involve the student in generating features of a diagram that are additional to those given in the problem statement and development of calculation paths. Chi et al. (1994) found that the students who were rated as frequent users of self-explanation (their high explainers) also drew more diagrams. The students' solution attempts in the current study were examined to identify instances of drawing of a diagram, labelling of parts of a diagram, extension of a diagram through construction and algebraic manipulations. Students were awarded one point for each instance of these features in a solution attempt.

Coding of post-training self-report questionnaires

The reports given by students in the final session concerning procedures that could be used to study new sections of their mathematics textbooks were analysed in order to identify study procedures that were related to the use of self-explanation. All instances that involved drawing of relationships between newly presented material and prior knowledge were identified in each student's response. The score given to each student represented the frequency of such reported procedures.

Path analysis procedures

The use of path analysis permits the examination of the self-explanation effect in several ways. First, the establishment of a path model allows examination of covarying relationships among the set of variables of interest in this study rather than limiting the analysis to consideration of a single independent variable at one time. In addition the latent variable approach enables more than a single indicator of a theoretical construct to be used, thereby providing for the possibility of developing a better representation of the theoretical construct. The measures used as indicators of each of the latent variables in this study are shown in Appendix 3.

The path analysis procedure used here is known as partial least squares path analysis (PLS) and employs the PLSPATH program that estimates path models with latent constructs measured by multiple indicators (see; Noonan and Wold, 1988; Sellin, 1986, Sellin, 1990; Sellin & Keeves, 1997). With PLS, the parameters of latent variable path models are estimated iteratively using least squares methods. PLS does not require stringent distributional assumptions, such as normality and independence of residuals. For these and other reasons, PLS is referred to as a "soft modelling" approach, which is useful specifically in research situations where exploratory model analyses under mild supplementary assumption are required (Sellin, 1990). Another feature of PLS is the explicit estimation of

latent variable scores. In order to evaluate PLS modelling results, it is possible to employ distribution-free statistical techniques, such as jackknifing. Applications of PLS have involved both the analysis of large path models with more than 100 observed variables and large data sets and the analysis of smaller models based on relatively small data sets with the aid of jackknife techniques (Sellin, 1990).

PLSPATH calculates a value for each latent, or theoretical, variable (LV) which is derived from the weights derived from analysis of a larger set of corresponding indicators, or manifest variables (MV). In PLS, as in covariance structure models, the analysis procedure is broken down into consideration of th letails of two submodels. The relationships between manifest and latent variables is described as the outer model. After the examination of outer model relationships the analysis moves to consideration of the relationships in the inner model, the relationships among the latent variables. These relationships are described by path coefficients. Relationships among variables in the inner model may be direct, from variable to variable, or indirect if the relationship is mediated by another variable. Estimates of direct and indirect effects are available in the PLS output.

The order of variables in the path model is established from left to right with the constructs on the left thought of as predicting constructs to the right. In the models discussed here the criterion variable being predicted by other latent variables is posttest performance. For a more comprehensive description of path modelling using partial least squares methods see Noonan and Wold (1988), Sellin (1986) and Sellin and Keeves (1997).

The analysis began with the fully recursive model shown in Figure 2, in which all possible paths from left to right were specified. The next runs of PLS involved trimming of the inner and outer models to identify the paths that best described the data. Paths that did not meet set criteria were dropped from the model. Decisions to keep or drop manifest variables and paths in the inner model were based on criteria described in Sellin and Keeves (1997) and Pedhazur, (1982).

Results

Student beliefs

The analysis of differences in mean scores of the groups on the beliefs questionnaire and on other measures discussed below were carried using independent t-tests. Where measures

derived from a task showed moderate degrees of correlation multivariate analysis of variance (MANOVA) tests were carried out followed by discriminant analysis (Weinfurt, 1995). Effect sizes for the comparisons between groups are indicated in the relevant tables.

The reliability of the total MBQ was 0.86, with subscale reliabilities ranging from 0.47 to 0.73. The means and standards deviations for the ratings of the two groups were very similar, for both the total score on the MBQ and for each of the subscale scores. The means of the groups did not differ significantly on any of these measures. The means for both groups were above the midpoint of the rating scale for each subscale, indicating that they had quite positive beliefs about themselves in relation to school mathematical activity. This may reflect their relative achievement standing in their year level at this school.

Study time

The use of the explanation questions did increase the length of time students spent in studying the new section of content. The mean completion time for the control group students was 20 minutes, with a standard deviation of 8.15 min. For the explanation group the corresponding were 26.02 minutes (6.20min.). This difference in study time was statistically significant: $t(41) = 2.81$, $p < .01$.

Verbal protocol analysis of study sessions

Table 1 shows the frequency of use of management, access and generation statements by the groups during their study of the new material and also the number of errors made during this study time. For this analysis the statements made by the self-explanation group students that involved repetition or direct responses to the set of self-explanation questions at the top of each page have been removed. Any differences between the groups on these measures therefore reflect the effects associated with use of the self-explanation questions.

The self-explanation group showed significantly higher frequency of use of each of the three types of process statements during their study sessions. In broad terms this group produced twice as much of each type of processing activity than did the students who were studying the same material using their typical procedures. Not only did the self-explanation group show more frequent management of their study processing but they were more active in drawing upon related geometry knowledge and in generating links with this related knowledge. The effect size for each comparison were large. The MANOVA analysis supported this interpretation, with management score having the highest discriminant loading.

Test performance

The pretest and overall posttest scores for the groups are shown in Table 2. Although the groups achieved very similar scores on the pretest of problem-related knowledge, their posttest performances differed substantially. The explanation group achieved a statistically significantly higher mean score on the posttest problems. When the scores for the two groups on the different components of the posttest are considered it is apparent that the major benefit associated with the explanation training occurred on the far transfer items, the items that required students to use both the target theorem and theorems not presented during study, or to construct new figures and to develop representations for word problems in order to generate a solution. The pattern of discriminant function loadings indicated that the far transfer score contributed most to the separation of the groups on the posttest.

Accessing of related theorem knowledge and elaboration activity

In this analysis the students' written solutions for items on the posttest were examined to identify instances of problem-related knowledge and the solution paths were categorised as being direct or indirect. The direct path score reflected the use by the student of additional theorems that provided the most economical solution path. The indirect path score indicated the frequency of use of theorem knowledge that was additional to that required for the most direct solution path. The results of this analysis are shown in Table 3.

As might be expected there was very little such access in the solutions for the training items. For both near and far transfer items, where additional theorems could be called upon in generation of a solution path, the solution attempts of the explanation group showed more frequent accessing of such theorems . On the near transfer items the difference between the groups reflected the greater use of indirect paths to solution by the self-explanation group. For the far transfer items both the indirect and direct path scores for the self-explanation group were significantly higher than those of the control group, although discriminant analysis showed that the direct path score contributed more to the separation of the groups than did the indirect score. In developing their solutions to the posttest items students in the self-explanation group were more active in use of related theorems that could used with the target theorem to develop a solution, though this accessing activity did not always involve use of the most economical solution path. The control group students showed a lower level of such access.

The frequency of elaboration activity of the groups also differed. The focus of interest here was the effect of self-explanation training on student's generation of additional features of the given diagram and calculation paths. The results of this analysis are shown in Table 4

Students in both groups used this form of constructive activity on each group of posttest items, although there was very little elaboration on the training items. The self-explanation group showed a higher frequency of elaboration on the other items, with the difference between the groups being most apparent on the near transfer items. This result suggests that the requirement to explain the new material encouraged more external constructive activity on the part of the student. It is not clear from this type of result whether this diagram elaboration, on its own, would have a beneficial effect on problem solving.

The analysis of students' written solutions indicated that the requirement to self-explain was associated with more problem-related activity in both searching for and using related knowledge and in the students' external constructive activity. The analysis of the verbal protocol data gathered during the students' study sessions permitted an examination of whether the patterns of study processes differed between the groups.

Domain-specific knowledge and strategy self-report

As was the case on the pretest, the groups did not differ in mean scores on the domain-specific knowledge test following completion of the training. These scores were useful in providing an estimate of the range of problem-relevant knowledge that was available to the groups before and after the self-explanation training. The lack of difference in these scores for the groups suggested that any difference in problem solving performance did not simply reflect that fact that the explanation group had acquired a wider body of knowledge than had the control group. Note here that these scores cannot be interpreted as showing that the organisation of these knowledge components was equivalent for the groups.

The final measure included in Table 2 was derived from the students' reports on methods that could be used to study new sections of content in mathematics. The scores here reflected the frequency with which these reports indicated that an explanation procedure could be used in this study task. This measure, therefore, provided both a check on the uptake of the explanation training and an estimate of group differences in metacognitive knowledge pertaining to study procedures in mathematics. The explanation group provided more reports that included the use of explanation procedures for this study task.

Path analysis

The hypothesised model of factors influencing problem solving performance as outlined in the PLSPATH conceptual framework consisted of 23 manifest variables which were combined into seven latent variables. During the model trimming process, preliminary PLSPATH runs revealed structural problems in nine cases. The manifest variables of

Mathab, Perfgoal, Lrnstyle, Stdrole, Mathtask, Quesaire, Acctrain, Gentrain and Trascore did not meet the set criteria and do were removed from the outer model equation. Only the final model for the combined scores on the near transfer and far transfer items is discussed here. In this model the outcome variable Performance is a weighted composite of two MVs, those for scores on the near and far transfer item sets. The path coefficients for the final model for this analysis is shown in Figure 2.

The value of R^2 is of particular interest for the outcome measure, Performance, because it suggests how well this latent variable is accounted for by other explanatory constructs in the model. It should be noted that the larger the variance on the criterion latent variable, the greater the likely explanatory power of the model.

First of all, it can be seen that in the Figure 2 there are no paths to the mediating variable of Beliefs from antecedents or from Beliefs to other constructs. The similarity of the profile of belief ratings for the students in this sample might well have artificially limited the influence of this variable in this analysis. Use of the self-explanation method had direct and indirect effects on other constructs. Use of the self-explanation method was associated with increased knowledge access and self management. Method had no direct effect on the Knowledge Generation or Performance, its influence on the later variable being mediated by knowledge access and knowledge generation processes. The total effect of method on Performance was 0.42. The strongest predictor of Performance in this model was Knowledge Generation with a direct effect of p=0.74. Indeed, Knowledge Generation was well predicted by other constructs (R^2=0.79). The R^2 for Performance indicated that 57 per cent of the variance on this construct was explained by the latent variables in the model, indicating that the model reflected the existing relationships to a substantial degree.

Prior Knowledge had effects on Performance through two paths: one was directly to Performance with path coefficient 0.13, the other one was through Knowledge Generation, though in both cases the effect was rather weak. The total effect of Prior Knowledge on Performance was 0.20, substantially lower than the total effect for Method. Knowledge Access and Knowledge Generation were very important mediating variables between Method and Performance in this model.

Discussion

The self-explanation effect observed in earlier research was also apparent in this investigation as students studied a new section of their geometry curriculum. The benefit

associated with the self-explanation training in the posttest was most obvious on the most complex problems, a result which paralleled that observed by Chi et al. (1994). The results obtained here suggest that the self-explanation is robust and likely to be worthy of the attention of mathematics teachers in classroom teaching. The effects observed here were obtained through use of a form of interaction with students that was less directive that had been used in previous research with high school students. This suggests that use of this form of study strategy could be used by classroom teachers without significant interference with other class activities. It must be noted, however, that the current study was not a classroom investigation and that the mean study times for the two groups were different. Although it was carried out in the school with the students' regular textbook materials, it did involve a researcher working individually with each student. So a study of teacher use of self-explanation effects in a regular classroom remains as a topic for further research. The greater time associated with the use of the self-explanation procedure remains a limitation of this study, although the findings of Renkl (1997) suggest that the equating of the study times of the two groups would not have removed all the self-explanation effect.

Of particular interest here is the finding that the effects of the self-explanation training were carried over from the period of training to a class test situation that was scheduled in the days following the original training. The analysis of the students' verbal protocols of study sessions showed that the self-explanation group engaged in much higher frequency of each of the three broad processing categories of access, generation and management. Here again the current results show a similar effect of self-explanation on processing activity during study as that described by Chi et al. (1994) and by Bielacyzc et al. (1995). This suggests that the self-explanation group provided themselves with increased chance of both establishing links among the elements of the new mathematical content between that new material and their existing domain-related knowledge. In the terms used by Mayer (1975) the self-explanation training seems to have increased the likelihood of both internal connectedness of components of a knowledge schema related to the new theorem and external connections between that schema and related schemas. The results of the path analysis suggest that these effects are crucial in describing how the requirement that students explain new material to themselves has an effect on their subsequent problem solving. Through its impact on knowledge access and generation activities, use of the self-explanation procedure might be seen to result in greater perturbation and focussing of the knowledge base relevant to the study topic. The strong indirect effects of method on performance in Figure 2 support this view.

Analysis of the students' written solution attempts during the posttest showed that the self-explanation group were both more active in accessing geometry knowledge related to the

test problems and in their elaboration of the details given in the problem statements and diagrams. In this case it is not study activity that is affected by the self-explanation training but use of the knowledge that has been studied on a previous occasion. This pattern of results suggests that the beneficial effect of self-explanation training is not limited to encoding and establishment of a knowledge representation, but is also operative, at least in the immediate future, when the students are called upon to activate and use this representation. Although the design of this study did not permit the making of claims about the quality of knowledge representations or of knowledge use, the increased accessing of related knowledge and the greater frequency of elaborative activity could be seen as giving the student a greater chance to identify and exploit knowledge that might be relevant to a particular problem.

The analysis of group differences did not reveal differences in the pattern of students' beliefs prior to the beginning of training and it remained for the path analysis to reveal if the strength of these beliefs had an impact on problem solving performance, either directly or indirectly. The process of selection of students for involvement in the study resulted in restriction of the range of strength of belief and so did not provide a useful test of the influence of this variable in the path model. With regard to prior knowledge effects the findings were similar to those of Chi et al. (1994), with one exception. The different forms of study procedure were not associated with marked differences in students' knowledge of the discrete components of knowledge assessed in the domain-specific knowledge section of the posttest. Even though there were differences in use of domain-related knowledge on the more difficult posttest items this was not associated with an obvious difference in the quantitative characteristics of the knowledge store at posttest, in terms of knowledge of the meaning of terms and theorems. Effectiveness of use of this knowledge was, however, different in the two groups.

The exceptional finding related to prior knowledge is the direct path from Prior Knowledge to Performance in the final path model. In this model prior knowledge is estimated by an additional variable, the class achievement score that represents a more comprehensive assessment of the students' past achievement in areas of the mathematics curriculum. In addition, the path analysis model allows for the effect of prior knowledge on performance to be estimated more directly, without the restriction imposed by the selection process for membership of the two groups that could have masked such an effect. Although the direct effect is not strong it does give some weight to the need to reconsider the interaction of prior knowledge and self-explanation processes. It was noted earlier that there is some reason to expect that prior knowledge would interact positively with self-explanation. If self-explanation processes could call upon richer, better organised knowledge bases, their use

might be expected to have greater impact than would their use with more impoverished stores of knowledge. This issue remains as an area for interesting research.

A final issue that requires comment is the pattern of relationships associated with self-management in the path model. On the basis of previous self-explanation research it might be expected that this variable would have a direct impact on performance, or one mediated through one of the knowledge processing variables. Chi et al. (1989) found that their most effective problem solvers both explained more and monitored their understanding more frequently than did less effective students. Additionally, Bielaczyc et al. (1995) found that training of both self-explanation and self-regulation strategies improved performance. In considering the significance of the current finding it is important to realise that there has not previously been an examination of the independent role of these different sets of processing activities. It is difficult in practice to separate the two activities and it could be seen that the self-explanation training questions did involve monitoring of understanding. So it is possible that there was an indirect effect of monitoring on performance. However, the measure used to form the self-management latent variable in the path analysis was distinct from the method variable, being derived from examination of students' solution attempts. The lack of a significant path does raise a question about the role of monitoring that requires further investigation. It could be that self-management has a differential effect on items that are more difficult. The recent findings of Renkl(1997), in which he made a distinction between positive and negative monitoring and observed different patterns of relationships between these forms of monitoring and the quality of self-explanation, adds weight to the need to examine the role of monitoring in a more sensitive manner.

References

Anderson, J. R. (1987). Skill acquisition. Compilation of weak-method problem solutions. *Psychological Review, 94,* 192 - 210.

Bielaczyc, K., Pirolli, P., & Brown, A. L. (1995). Training in self-explanation and self-regulation strategies: Investigating the effects of knowledge acquisition activities on problem solving. *Cognition and Instruction, 13,* 221 - 252.

Chi, M. T. H., Bassock, M. Lewis, M. W., Reimann, P. & Glaser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science, 13,* 145 - 182.

Chi, M. T. H., & VanLehn, K. (1991). The content of physics self-explanations. *The Journal of the Learning Sciences, 1,* 69 - 105.

Chi, M. T. H., De Leeuw, N., Chiu, M-H. & LaVancher, C. (1994). Eliciting self-explanations improves understanding. *Cognitive Science, 18,* 439 - 477.

Chinnappan, M. & Lawson, M. J. (1994). School geometry: Focus on knowledge organisation. *Australian Mathematics Teacher, 50,* 14 - 18.

Ericsson, K. A., & Simon, H. A. (1993). *Protocol analysis: Verbal reports as data,.* (revised edition). Cambridge, MA: Bradford Books/MIT Press.

Fergusson-Heesler, M. G. M. & de Jong, T. (1990). Studying physics texts: Differences in study processes between good and poor solvers. *Cognition and Instruction, 7,* 41 - 54.

Glaser, R. (1994). Learning theory and instruction. In G. d'Ydewalle, P. Eelen, & P. Bertelson (Eds.) *International perspectives on psychological science, Vol. 2: The state of the art* (pp. 341 - 357). Hove, UK.: Lawrence Erlbaum Associates.

Hayes, A. (1993). *Student beliefs and mathematical performance on problem solving tasks.* Unpublished honours thesis, Flinders University of South Australia, Adelaide.

Hays, W.L. (1994). *Statistics* (Fifth edition). Orlando, FLA: Harcourt Brace.

Keeves, J. P. (1988). Multivariate analysis. In J. P. Keeves (Ed.), *Educational research, methodology, and measurement: An international handbook*. Oxford: Pergamon Press.

Lawson, M. J. & Chinnappan, M (1994). Generative activity during geometry problem solving: Comparison of the performance of high-achieving and low-achieving students. *Cognition and Instruction, 12*, 61-93.

Mayer, R. E. (1975). Information processing variables in learning to solve problems. *Review of Educational Research, 45*, 525 - 541.

McLeod, D. B., & Adams, V. M. (1989). *Affect and mathematical problem solving*. New York: Springer-Verlag.

Noonan, R.D., & Wold, H. (1988). Partial least squares path analysis. In J.P.Keeves (Ed.), *Educational research , methodology, and measurement: An international handbook*. Oxford: Pergamon Press.

Pedhazur, E. J. (1982). *Multiple regression in behavioral research: Explanation and prediction*. Holt, Rinehart, and Winston, New York.

Pirolli, P., & Bielaczyc, K. (1989). Empirical analyses of self-explanation and transfer in learning to program. In G. M. Olson & E. E. Smith (Eds.), *Proceedings of the 11th Conference of the Cognitive Science Society* (pp. 450 -457). Hillsdale, NJ: Lawrence Erlbaum Associates.

Pirolli, P., & Recker, M. (1994). Learning strategies and transfer in the domain of programming. *Cognition and Instruction, 12*, 235 - 275.

Prawat, R. (1989). Promoting access to knowledge, strategy and disposition in students. *Review of Educational Research*, 59, 1-42.

Recker, M. & Pirolli, P. (1995). Modeling individual differences in students' learning strategies. *The Journal of the Learning Sciences, 4*, 1-38.

Renkl, A. (1997). Learning from worked-out examples: a study on individual differences. *Cognitive Science, 21*, 1 - 29.

Schunk, D. H. (1983). Progress self-monitoring: Effects on children's self-efficacy and achievement. *Journal of Experimental Education, 51*, 89 - 93.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. New york: Academic Press.

Sellin, N. (1986). Aspiration, motivation, and achievement. Different methods of analysis and analysis and different results. *International Journal of Educational Research, 10*, 117-243.

Sellin, N. (1990). *PLSPATH version 3.01. Program manual*. Hamburg, Germany.

Sellin, N. & Keeves, J. P. (1997). Path analysis with latent variables. In J. P. Keeves(Ed.). *Educational research, methodology and measurement* (2nd ed.). (pp. 633 - 640). Cambridge, UK: Pergamon Press.

VanLehn, K., Jones, R. M. & Chi, M. T. H. (1992). A model of the self-explanation effect. *The Journal of the Learning Sciences, 2*, 1 - 59.

Weinfurt, K. (1995). Multivariate analysis of variance. In L. G. Grimm and P. R. Yarnold (Eds.). *Reading and understanding multivariate statistics* (pp.245 - 276). Washington, DC: American Psychological Association.

Woloshyn, V. E., Pressley, M. & Schneider, W. (1992). Elaborative-interrogation and prior-knowledge effects on learning of facts. *Journal of Educational Psychology, 84*, 115 - 124.

Zimmerman, B. J. (1989). A social cognitive view of self-regulated academic learning. *Journal of Educational Psychology, 81*, 329 -339.

Table 1. Frequency of statements on verbal protocol measures during study.

	Self-explanation group		Control group			
	M	SD	M	SD	t-value	Effect size[a]
Measure						
Management statements	57.67	12.87	21.39	11.33	10.27**	1.66
Access statement	34.88	9.27	20.13	5.90	6.53**	1.37
Generation statements	41.54	17.78	21.83	13.22	4.32**	1.07
Error	4.33	2.73	4.78	4.82	-.39	0.12

** t-value $p < .01$

[a] Effect sizes calculated following Hays (1994, p. 411).

Table 2. Pretest and posttest performance for explanation and control groups.

	Explanation Group (n = 24)		Control Group (n = 23)			
Measure (possible score)	M	SD	M	SD	t-value	Effect size[a]
Pretest	25.58	3.37	25.74	2.99	0.17	0.05
Posttest: total	31.58	8.45	25.78	8.89	2.29*	0.64
Training (2)	2.00	0	1.91	0.42	1.00	0.30
Near transfer (17)	14.63	3.85	13.30	4.52	1.08	0.31
Far transfer (25)	14.96	6.14	10.57	6.07	2.47*	0.68
Domain-specific knowledge	9.75	1.07	9.43	0.56	0.75	0.21
Strategy self-report score	0.79	0.72	0.30	0.56	2.60*	0.71

* $p < 05$

[a] Effect sizes calculated following Hays (1994, p. 411).

Table 3. Frequency of access of additional theorems in written posttest solutions.

		Explanation group (n=24)		Control group (n=23)		t-value	Effect size[a]
Items	Measure	M	SD	M	SD		
Training	Total	0.58	2.86	0	0	1.00	0.28
	Indirect path	0.58	2.86	0	0	1.00	0.29
	Direct path	0	0	0	0	0	0
Near transfer	Total	10.33	4.28	6.35	3.14	3.65**	0.94
	Indirect path	6.21	4.50	1.91	2.83	3.93**	0.99
	Direct path	4.13	1.73	4.43	2.47	0.50	0.15
Far transfer	Total	14.33	7.53	9.09	5.94	2.66*	0.73
	Indirect path	8.29	5.80	5.26	4.11	2.07*	0.58
	Direct path	6.04	3.11	3.83	2.65	2.65*	0.72

* t-value $p < .05$ ** t-value $p < .01$

[a] Effect sizes calculated following Hays (1994, p. 411).

Table 4. Frequency of elaboration activity in posttest solution attempts.

	Self-explanation group (n=24)		Control group (n=23)			
Test Items	M	SD	M	SD	t-value	Effect size[a]
Posttest Total	47.13	18.10	29.87	17.57	3.32**	0.88
Training	1.63	4.41	0.52	0.79	1.21	0.34
Near transfer	21.00	9.69	12.65	7.16	3.37**	0.88
Far transfer	24.50	10.23	16.70	11.87	2.41*	0.67

* t-value $p < .05$ ** t-value $p < .01$

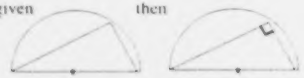
[a] Effect sizes calculated following Hays (1994, p. 411).

Figure 1. The complete text of the study material.

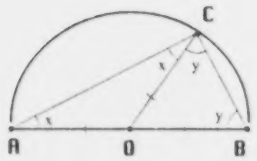
Angle in a semi-circle theorem

The angle in a semi-circle is a right-angle

i.e., given then



Proof:



Let C be any point on a semi-circle (other than at A or B) where AB is its diameter.

Let ∠ CAB be x and ∠ CBA be y. Join $\overline{OC}$.

In ΔOAC,

OA = OC {equal radii}

∴ ΔOAC is isosceles

∴∠ACO =∠ CAO = x {equal base angles}

Likewise, ΔBOC is isosceles and ∴∠ BCO = y.

In ΔABC,

x + (x + y) + y = 180° {sum of angles of triangle}

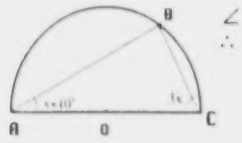
∴ 2x + 2y = 180°

∴ x + y = 90°

Hence angle ACB is a right-angle.

Example 1:

Find the value of x :



∠ ABC measures 90° {angle in a semicircle}

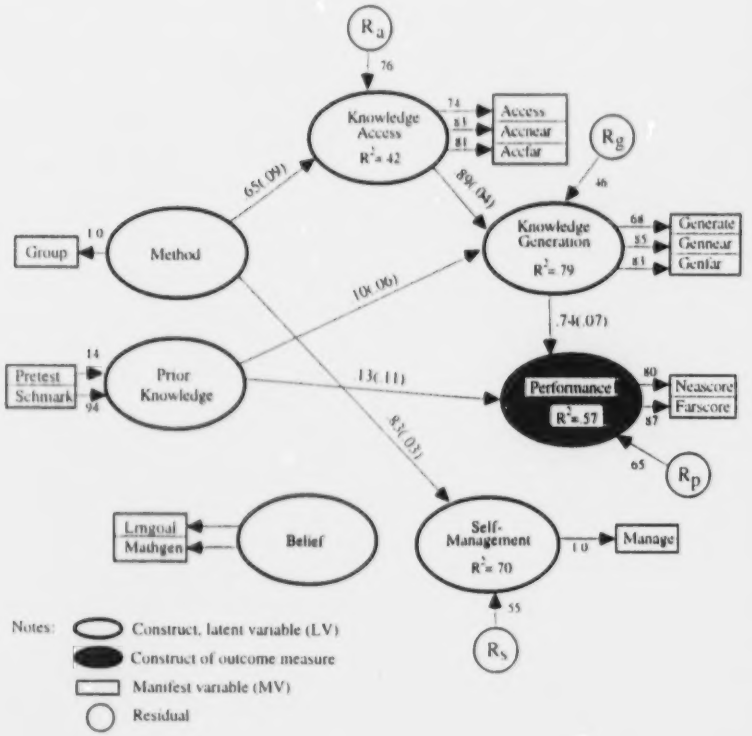
∴ x + 10° + 3x + 90° = 180° {sum of angles of triangle}

∴ 4x + 100° = 180°

∴ 4x = 80°

∴ x = 20°

Figure 2. Final path model for combined near and far transfer posttest performance. (n = 47)



BEST COPY AVAILABLE

Appendix 1. Code labels and descriptions of three major categories of events

Self-Management		
M1	Re-reading	Re-reading a text statement, which includes paraphrasing where nothing new is added.
M2	Evaluation of given information as understood	Showing certain information in the text as understood, or identifying a difficulty
M3	Evaluation of given information as area of difficulty	Showing certain information in the text as not understood or difficult.
M4	Restating difficulty	Restating a difficulty that has been mentioned earlier
M5	Restating hypothesising	Retrieving a hypothesis that has been mentioned earlier.
M6	Checking	Reassessing of the task, information, computation or hypothesising
M7	Review	The expression of acceptance or confirmation of previously completed activity.
M8	Identification of given information as new information	Identification of given information as a new section information from last page

Knowledge Access		
A1	Identification of implicit information	Identifying all different kinds of implicit information
A2	Retrieval of geometrical form, term or symbol	Retrieving of a geometrical feature, symbol or term
A3	Retrieval of theorem, proof or other knowledge	Retrieving a studied theorem, proof, other geometrical theorems, or other geometrical knowledge.
A4	Retrieval of generated information	Retrieving generated information that has been mentioned earlier
Knowledge Generation		
G1	Diagram	Drawing of a diagram, extensions to given or drawn diagram, or labelling of a diagram.
G2	Generation of information	Generation of new knowledge components
G3	Hypothesising	Mentioning an adequate idea that is used as basis for further investigation.
G4	Reasoning	Engaging in elementary reasoning in the course of identifying given or generated information
G5	Justification	Showing adequate reason to clarify information
G6	Application	Using procedural knowledge

G7	Algebraic manipulation/ Computation	Simplification of an equation or carrying out of operations

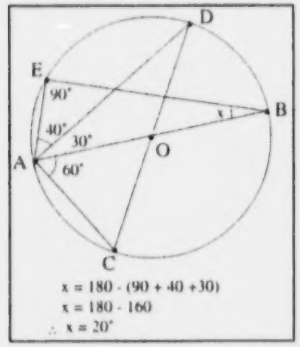
Error

E Error	Error in generation, reasoning, omission and reading	

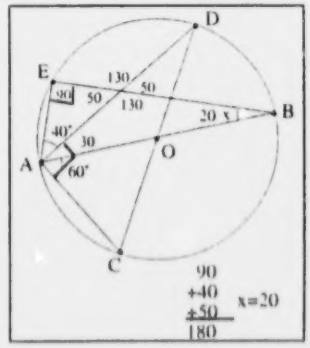
Note: Key question re-reading (013) not including in M1

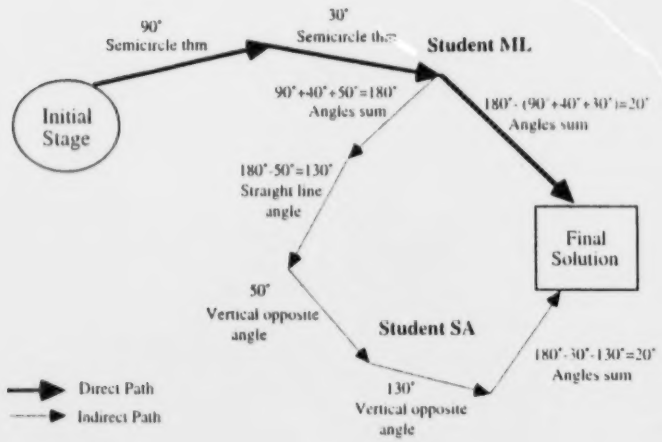
Appendix 2. Illustration of direct and indirect paths in written solutions of two students.

Student ML



Student SA





Appendix 3. Description of manifest and latent variables used in path analysis.

Latent Variable	Manifest Variable
Method	Self-Explanation Method
	identifying whether the student is in SE group or control group
	• Group
Prior Knowledge	Student's Prior Knowledge
	measuring student's previous mathematics knowledge
	• Score on Pretest (Pretest)
	• School Mark (Schmark)
Beliefs	Student's beliefs
	measuring student's mathematical beliefs using a questionnaire
	• Nature of Mathematics Ability (Mathab)
	• Performance Goal (Perfgoal)
	• Learning Goal (Lrngoal)
	• Learning Style (Lrnstyle)
	• Student Role (stdrole)
	• Nature of Mathematics-Gen (Mathgen)
	• Nature of Mathematics-Task (Mathtask)

Self Management	Student's Self-Management
	measuring the number of statements or actions such as: rereading of the text; identification of a text element as new; monitoring of comprehension; statement or re-stating of difficulties in comprehension; checking of the goal, of the accuracy of reasoning, of calculations; evaluation of the solution.
	• Protocol Self-Management (Manage)
Knowledge Access	Student's Knowledge Access
	measuring the number of statement or action which involved identification, retrieval of names or meanings of geometrical features or of theorems.
	• Protocol Knowledge Access (Access)
	• Knowledge Access from Training Items (Acctrain)
	• Knowledge Access from Near Transfer Items (Accnear)
	• Knowledge Access from Far Transfer Items (Accfar)

Knowledge Generation	Student's Knowledge Generation *measuring the number of statements or actions which involved drawing, labelling or extension of a diagram, calculations and algebraic manipulations reasoning or hypothesising or justifying that involved application of the target theorem or related theorems that had been previously accessed* • Protocol Knowledge Generation (Generate) • Self-report Questionnaire (Quesaire) • Knowledge elaboration from Training Items (Gentrain) • Knowledge elaboration from Near Transfer Items (Gennear) • Knowledge elaboration from Far Transfer Items (Genfar)
Performance	Student's Problem Solving Performance *measuring student's mathematics performance using a posttest which includes training, near transfer and far transfer items* • Total Score of Training Items (Trascore) • Total Score of Near Transfer Items (Neascore) • Total Score of Far Transfer Items (Farscore)